

Particle phenomenology and Maldacena

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Abstract. A brief review is offered of employing Maldacena's AdS/CFT correspondence in attempting to identify a model which extends to higher energy the standard model of particle phenomenology.

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INTRODUCTION

It is an honor to speak at a meeting in his home city commemorating the tenth anniversary of Maldacena's paper[1] on the AdS/CFT correspondence which has acquired five thousand citations in its first ten years. Its popularity stems from the diversity of its applications. As merely one example, it attracted me back to string theory in 1998 after years away by a derivative conjecture[2], admittedly going beyond what Maldacena stated, that a non-supersymmetric finite $SU(N)$ gauge theory can be conformally invariant at high energies, a conjecture which has the disadvantage of being technically difficult to study. A no-go theorem[3] exists but uses questionable assumptions.

ADS/CFT AND STANDARD MODEL

The physical idea is to consider the standard model from a perspective of an energy of a few TeV or more where quark and lepton masses, the QCD and weak scales, become negligible and the theory is classically scale invariant. Quantum mechanically it is not so because of beta functions and anomalous dimensions which generically do not vanish. The idea is to enrich the theory in such a way that it becomes also quantum mechanically conformally invariant at high energies.

A recent comprehensive review[4] contains a historical section which indicates the special role played by the paper[1] in phenomenology beyond the standard model and it seems appropriate to reproduce it in its entirety here.

IMPACT OF MALDACENA'S PAPER ON PARTICLE PHENOMENOLOGY

Particle phenomenology is in an especially exciting time, mainly because of the anticipated data in a new energy regime expected from the Large Hadron Collider (LHC), to

be commissioned at the CERN Laboratory in 2007. This new data is long overdue. The Superconducting Supercollider (SSC) could have provided such data long ago were it not for its political demise in 1993.

Except for the remarkable experimental data concerning neutrino masses and mixings which has been obtained since 1998, particle physics has been data starved for the last thirty years. The standard model invented in the '60s and '70s has been confirmed and reconfirmed. Consequently, theory has ventured into speculative areas such as string theory, extra dimensions and supersymmetry. While these ideas are of great interest and theoretically consistent there is no *direct* evidence from experiment for them. Here we describe a more recent, post 1998, direction known as conformality. First, to set the stage, we shall discuss why the conformality approach which is, in our opinion, competitive with the other three approaches, remained unstudied for the twenty years up to 1998.

A principal motivation underlying model building, beyond the standard model, over the last thirty years has been the *hierarchy problem* which is a special case of *naturalness*. This idea stems from Wilson[5] in the late '60s. The definition of naturalness is that a theory should not contain any unexplained very large (or very small in the inverse) dimensionless numbers. The adjustment needed to achieve such naturalness violating numbers is called *fine tuning*. The naturalness situation can be especially acute in gauge field theories because even after fine tuning at tree level, *i.e.*, the classical lagrangian, the fine tuning may need to be repeated an infinite number of times order by order in the loop expansion during the renormalization process. While such a theory can be internally consistent it violates naturalness. Thus naturalness is not only an aesthetic criterion but one which the vast majority of the community feel must be imposed on any acceptable extension of the standard model; ironically, one exception is Wilson himself [6].

When the standard model of Glashow[7] was rendered renormalizable by appending the Higgs mechanism[8, 9] it was soon realized that it fell into trouble with naturalness, specifically through the hierarchy problem. In particular, the scalar propagator has quadratically divergent radiative corrections which imply that a bare Higgs mass m_H^2 will be corrected by an amount Λ^2/m_H^2 where Λ is the cut off scale corresponding to new physics. Unlike logarithmic divergences, which can be absorbed in the usual renormalization process, the quadratic divergences create an unacceptable fine tuning: for example, if the cut off is at the conventional grand unification scale $\Lambda \sim 10^{16}$ GeV and $m_H \sim 100$ GeV, we are confronted with a preposterous degree of fine tuning to one part in 10^{28} .

As already noted, this hierarchy problem was stated most forcefully by Wilson who said, in private discussions, that scalar fields are forbidden in gauge field theories. Between the late '60s and 1974, it was widely recognized that the scalar fields of the standard model created this serious hierarchy problem but no one knew what to do about it.

The next big progress to the hierarchy problem came in 1974 with the invention[10] of supersymmetry. This led to the Minimally Supersymmetric Standard Model (MSSM) which elegantly answered Wilson's objection since quadratic divergences are cancelled between bosons and fermions, with only logarithmic divergences surviving. Further it was proved [11, 12] that the MSSM and straightforward generalizations were the unique

way to proceed. Not surprisingly, the MSSM immediately became overwhelmingly popular. It has been estimated [13] that there are 35,000 papers existing on supersymmetry, more than an average of one thousand papers per year since its inception. This approach continued to seem "unique" until 1998. Since the MSSM has over one hundred free parameters, many possibilities needed to be investigated and exclusion plots constructed. During this period, two properties beyond naturalness rendered the MSSM even more appealing: an improvement in unification properties and a candidate for cosmological dark matter.

Before jumping to 1998, it is necessary to mention an unconnected development in 1983 which was the study of Yang-Mills theory with extended $\mathcal{N} = 4$ supersymmetry (the MSSM has $\mathcal{N} = 1$ supersymmetry). This remarkable theory, though phenomenologically quite unrealistic as it allows no chiral fermions and all matter fields are in adjoint representations, is finite [14, 15, 16] to all orders of perturbation theory and conformally invariant. Between 1983 and 1997, the relationship between the $\mathcal{N} = 4$ gauge theory and either string theory, also believed to be finite, or the standard model remained unclear.

The perspective changed in 1997-98 initially through the insight of Maldacena[1] who showed a *duality* between $\mathcal{N} = 4$ gauge theory and the superstring in ten spacetime dimensions. Further the $\mathcal{N} = 4$ supersymmetry can be broken by orbifolding down to $\mathcal{N} = 0$ models with no supersymmetry at all. It was conjectured [2] by one of the authors in 1998 that such nonsupersymmetric orbifolded models can be finite and conformally invariant, hence the name conformality.

Conformality models have been investigated far less completely than supersymmetric ones but it is already clear that supersymmetry is "not as unique" as previously believed. No-go theorems can have not only explicit assumptions which need to be violated to avoid the theorem but unconscious implicit assumptions which require further progress even to appreciate: in 1975 the implicit assumption was that the gauge group is simple, or if semi-simple may be regarded as a product of theories each with a simple gauge group. Naturalness, by cancellation of quadratic divergences, accurate unification and a dark matter candidate exist in conformality.

It becomes therefore a concern that the design of the LHC has been influenced by the requirement of testing the MSSM. The LHC merits an investment of theoretical work to check if the LHC is adequately designed to test conformality which now seems equally as likely as supersymmetry, although we fully expect the detectors ATLAS and CMS to be sufficiently all purpose to capture any physics beyond the standard model at the TeV scale.

QUIVER GAUGE THEORIES

Quiver gauge theories possess a gauge group which is generically a product of $U(N_i)$ factors with matter fields in bifundamental representations. They have been studied in the physics literature since the 1980s where they were used in composite model building. They have attracted much renewed attention because of their natural appearance in the duality between superstrings and gauge theories.

The best known such duality gives rise to a highly supersymmetric ($\mathcal{N} = 4$) gauge theory with a single $SU(N)$ gauge group with matter in adjoint representations. In this case one can drop with impunity the $U(1)$ of $U(N)$ because the matter fields are uncharged under it. In the quiver theories with less supersymmetry ($\mathcal{N} \leq 2$) it is usually necessary to keep such $U(1)$ s.

Quiver gauge theories are tailor made for particle physics model building. While an $SU(N)$ gauge theory is typically anomalous in for arbitrary choice of fermions, choosing the fermions to lie in a quiver insures anomaly cancelation. Furthermore the fermions in a quiver arrange themselves in bifundamental representations of the product gauge group. This nicely coincides with the fact that all known fundamental fermions are in bifundamental, fundamental, or singlet representations of the gauge group. The study of quiver gauge theories goes back to the earliest days of gauge theories and the standard model. Other notable early examples are the Pati-Salam model and the trinification model. A vast literature exists on this subject, but we will concentrate on post AdS/CFT conjecture quiver gauge theory work [4] Starting from $AdS_5 \times S^5$ we only have an $SU(N)$ $\mathcal{N} = 4$ supersymmetric gauge theory. In order to break SUSY and generate a quiver gauge theory there are several options open to us. Orbifolds [4], conifolds [4] and orientifolds [4] have all played a part in building quiver gauge theories.

It is important to note that although the duality with superstrings is a significant guide to such model building, and it is desirable to have a string dual to give more confidence in consistency, we shall focus on the gauge theory description in the approach to particle phenomenology, as there are perfectly good quiver gauge theories that have yet to be derived from string duality.

ORBIFOLDING

The simplest superstring - gauge duality arises from the compactification of a Type IIB superstring on the cleverly chosen manifold

$$AdS_5 \times S^5$$

which yields an $\mathcal{N} = 4$ supersymmetry which is an especially interesting gauge theory which has been intensively studied and possesses remarkable properties of finiteness and conformal invariance for all values of N in its $SU(N)$ gauge group. By "conformality", we shall mean conformal invariance at high energy, also for finite N .

For phenomenological purposes, $\mathcal{N} = 4$ is too much supersymmetry. Fortunately it is possible to breaking supersymmetries and hence approach more nearly the real world, with less or no supersymmetry in a conformality theory.

By factoring out a discrete (either abelian or nonabelian) group and composing an orbifold:

$$S^5/\Gamma$$

one may break $\mathcal{N} = 4$ supersymmetry to $\mathcal{N} = 2$, 1, or 0. Of special interest is the $\mathcal{N} = 0$ case.

We may take an abelian $\Gamma = Z_p$ (non-abelian cases will also be considered in this review) which identifies p points in a complex three dimensional space \mathcal{C}_3 .

The rules for breaking the $\mathcal{N} = 4$ supersymmetry are:

If Γ can be embedded in an $SU(2)$ of the original $SU(4)$ R-symmetry, then

$$\Gamma \subset SU(2) \Rightarrow \mathcal{N} = 2.$$

If Γ can be embedded in an $SU(3)$ but not an $SU(2)$ of the original $SU(4)$ R-symmetry, then

$$\Gamma \subset SU(3) \Rightarrow \mathcal{N} = 1.$$

If Γ can be embedded in the $SU(4)$ but not an $SU(3)$ of the original $SU(4)$ R-symmetry, then

$$\Gamma \subset SU(4) \Rightarrow \mathcal{N} = 0.$$

In fact to specify the embedding of $\Gamma = Z_p$ we need to identify three integers (a_1, a_2, a_3) :

$$\mathcal{C}_3 : (X_1, X_2, X_3) \xrightarrow{Z_p} (\alpha^{a_1} X_1, \alpha^{a_2} X_2, \alpha^{a_3} X_3)$$

with

$$\alpha = \exp\left(\frac{2\pi i}{p}\right)$$

The Z_p discrete group identifies p points in \mathcal{C}_3 . The N converging D3-branes meet on all p copies, giving a gauge group: $U(N) \times U(N) \times \dots \times U(N)$, p times. The matter (spin-1/2 and spin-0) which survives is invariant under a product of a gauge transformation and a Z_p transformation.

There is a convenient diagrammatic way to find the result from a "quiver." One draws p points and arrows for a_1, a_2, a_3 .

For a general case, the scalar representation contains the bifundamental scalars

$$\sum_{k=1}^3 \sum_{i=1}^p (N_1, \bar{N}_{i \pm a_k})$$

For fermions, one must first construct the **4** of R-parity $SU(4)$, isomorphic to the isometry $SO(6)$ of the S^5 . From the $a_k = (a_1, a_2, a_3)$ one constructs the 4-spinor $A_\mu = (A_1, A_2, A_3, A_4)$:

$$A_1 = \frac{1}{2}(a_1 + a_2 + a_3)$$

$$A_2 = \frac{1}{2}(a_1 - a_2 - a_3)$$

$$A_3 = \frac{1}{2}(-a_1 + a_2 - a_3)$$

$$A_4 = \frac{1}{2}(-a_1 - a_2 + a_3)$$

These transform as $\exp\left(\frac{2\pi i}{p}A_\mu\right)$ and the invariants may again be derived by a different quiver diagram.

Note that these lines are oriented, as is necessary to accommodate chiral fermions. Specifying the four A_μ is equivalent (there is a constraint that the four add to zero, mod p) to fixing the three a_k and group theoretically is more fundamental.

In general, the fermion representation contains the bifundamentals

$$\sum_{\mu=1}^4 \sum_{i=1}^p (N_i, \tilde{N}_{i+A_\mu})$$

When one of the A_μ s is zero, it signifies a degenerate case of a bifundamental comprised of adjoint and singlet representations of one $U(N)$.

CONFORMALITY PHENOMENOLOGY

In attempting to go beyond the standard model, one outstanding issue is the hierarchy between GUT scale and weak scale which is 14 orders of magnitude. Why do these two very different scales exist? Also, how is this hierarchy of scales stabilized under quantum corrections? Supersymmetry answers the second question but not the first.

The idea is to approach hierarchy problem by Conformality at a TeV Scale. We will show how this is possible including explicit examples containing standard model states.

In some sense conformality provides more rigid constraints than supersymmetry. It can predict additional states at TeV scale, while there can be far fewer initial parameters in conformality models than in SUSY models. Conformality also provides a new approach to gauge coupling unification. It confronts naturalness and provides cancellation of quadratic divergences. The requirements of anomaly cancellations can lead to conformality of U(1) couplings.

There is a viable dark matter candidate, and proton decay can be consistent with experiment.

What is the physical intuition and picture underlying conformality? Consider going to an energy scale higher than the weak scale, for example at the TeV scale. Quark and lepton masses, QCD and weak scales small compared to TeV scale. They may be approximated by zero. The theory is then classically conformally invariant though not at the quantum level because the standard model has non-vanishing renormalization group beta functions and anomalous dimensions. So this suggests that we add degrees of freedom to yield a gauge field theory with conformal invariance. There will be 't Hooft's naturalness since the zero mass limit increases symmetry to conformal symmetry.

We have no full understanding of how four-dimensional conformal symmetry can be broken spontaneously so breaking softly by relevant operators is a first step. The theory is assumed to be given by the action:

$$S = S_0 + \int d^4x \alpha_i O_i \quad (1)$$

where S_0 is the action for the conformal theory and the O_i are operators with dimension below four (*i.e.* relevant) which break conformal invariance softly.

The mass parameters α_i have mass dimension $4 - \Delta_i$ where Δ_i is the dimension of O_i at the conformal point.

Let M be the scale set by the parameters α_i and hence the scale at which conformal invariance is broken. Then for $E \gg M$ the couplings will not run while they start running for $E < M$. To solve the hierarchy problem we assume M is near the TeV scale.

EXPERIMENTAL EVIDENCE FOR CONFORMALITY

Consider embedding the standard model gauge group according to:

$$SU(3) \times SU(2) \times U(1) \subset \bigotimes_i U(Nd_i)$$

Each gauge group of the SM can lie entirely in a $SU(Nd_i)$ or in a diagonal subgroup of a number thereof.

Only bifundamentals (including adjoints) are possible. This implies no $(8,2), (3,3)$, etc. A conformality restriction which is new and satisfied in Nature! The fact that the standard model has matter fields all of which can be accommodated as bifundamentals is experimental evidence for conformality.

No $U(1)$ factor can be conformal in perturbation theory and so hypercharge is quantized through its incorporation in a non-abelian gauge group. This is the "conformality" equivalent to the GUT charge quantization condition in *e.g.* $SU(5)$. It can explain

the neutrality of the hydrogen atom. While these are postdictions, the predictions of the theory are new particles, perhaps at a low mass scale, filling out bifundamental representations of the gauge group that restore conformal invariance. The next section will begin our study of known quiver gauge theories from orbifolded $AdS^5 \times S^5$.

TABULATION OF THE SIMPLEST ABELIAN QUIVERS

We consider the compactification of the type-IIB superstring on the orbifold $AdS_5 \times S^5/\Gamma$ where Γ is an abelian group $\Gamma = Z_p$ of order p with elements $\exp(2\pi i A/p)$, $0 \leq A \leq (p-1)$.

The resultant quiver gauge theory has \mathcal{N} residual supersymmetries with $\mathcal{N} = 2, 1, 0$ depending on the details of the embedding of Γ in the $SU(4)$ group which is the isotropy of the S^5 . This embedding is specified by the four integers $A_m, 1 \leq m \leq 4$ with

$$\sum_m A_m = 0 \mod p \quad (2)$$

which characterize the transformation of the components of the defining representation of $SU(4)$. We are here interested in the non-supersymmetric case $\mathcal{N} = 0$ which occurs if and only if all four A_m are non-vanishing.

Table I. All abelian quiver theories with $\mathcal{N} = 0$ from Z_2 to Z_5 .

	p	A_m	a_i	scal bdfs	scal adjs	chir frms	SM
1	2	(1111)	(000)	0	6	No	No
2	3	(1122)	(001)	2	4	No	No
3	4	(2222)	(000)	0	6	No	No
4	4	(1133)	(002)	2	4	No	No
5	4	(1223)	(011)	4	2	No	No
6	4	(1111)	(222)	6	0	Yes	No
7	5	(1144)	(002)	2	4	No	No
8	5	(2233)	(001)	2	4	No	No
9	5	(1234)	(012)	4	2	No	No
10	5	(1112)	(222)	6	0	Yes	No
11	5	(2224)	(111)	6	0	Yes	No

CHIRAL FERMIONS

The gauge group is $U(N)^p$. The fermions are all in the bifundamental representations

$$\sum_{m=1}^{m=4} \sum_{j=1}^{j=p} (N_j, \bar{N}_{j+A_m}) \quad (3)$$

which are manifestly non-supersymmetric because no fermions are in adjoint representations of the gauge group. Scalars appear in representations

$$\sum_{i=1}^{i=3} \sum_{j=1}^{j=p} (N_j, \bar{N}_{j \pm a_i}) \quad (4)$$

in which the six integers $(a_i, -a_i)$ characterize the transformation of the antisymmetric second-rank tensor representation of $SU(4)$. The a_i are given by $a_1 = (A_2 + A_3)$, $a_2 = (A_3 + A_1)$, and $a_3 = (A_1 + A_2)$.

It is possible for one or more of the a_i to vanish in which case the corresponding scalar representation in the summation in Eq.(4) is to be interpreted as an adjoint representation of one particular $U(N)_j$. One may therefore have zero, two, four or all six of the scalar representations, in Eq.(4), in such adjoints. One purpose of the present article is to investigate how the renormalization properties and occurrence of quadratic divergences depend on the distribution of scalars into bifundamental and adjoint representations.

Note that there is one model with all scalars in adjoints for each even value of p . For general even p the embedding is $A_m = (\frac{p}{2}, \frac{p}{2}, \frac{p}{2}, \frac{p}{2})$. This series by itself forms the complete list of $\mathcal{N} = 0$ abelian quivers with all scalars in adjoints.

To be of more phenomenological interest the model should contain chiral fermions. This requires that the embedding be complex: $A_m \not\equiv -A_m \pmod{p}$. It will now be shown that for the presence of chiral fermions all scalars must be in bifundamentals.

The proof of this assertion follows by assuming the contrary, that there is at least one adjoint arising from, say, $a_1 = 0$. Therefore $A_3 = -A_2 \pmod{p}$. But then it follows from Eq.(2) that $A_1 = -A_4 \pmod{p}$. The fundamental representation of $SU(4)$ is thus real and fermions are non-chiral.

The converse also holds: If all $a_i \neq 0$ then there are chiral fermions. This follows since by assumption $A_1 \neq -A_2$, $A_1 \neq -A_3$, $A_1 \neq -A_4$. Therefore reality of the fundamental representation would require $A_1 \equiv -A_1$ hence, since $A_1 \neq 0$, p is even and $A_1 \equiv \frac{p}{2}$; but then the other A_m cannot combine to give only vector-like fermions.

It follows that:

In an $\mathcal{N} = 0$ quiver gauge theory, chiral fermions are possible if and only if all scalars are in bifundamental representations.

OTHER DEVELOPMENTS

The orbifold model building has been extended to non-abelian finite groups including the analysis of every such group of order $g \leq 31$. This can give rise to more general unifying gauge groups like $SU(4) \times SU(2) \times SU(2)$ and to interesting such chiral models.

Grand unification models with TeV scale unification have attracted attention because the unification of the couplings occurs in a novel fashion associated with the group embeddings of $SU(3)$, $SU(2)$ and $U(1)$. Such unification is precisely accurate, as much so or more than supersymmetric grand unification.

Quadratic divergences in the scalar two-point function are canceled due to a generalization of supersymmetry, named misaligned supersymmetry whose explicit realization is a challenging open question.

There is an attractive dark matter candidate called the LCP or Lightest Conformality Particle.

For more details about these further developments we refer the reader to the review article listed[4].

CONGRATULATIONS

When the paper [1] first appeared in November 1997, having not recently worked on string theory, I remained unaware of it until July 1998 when, visiting CERN, almost every theory seminar was about AdS/CFT. There had recently been a string conference in Santa Barbara where almost all talks were on AdS/CFT. Participants there even danced to an AdS/CFT song[17]!

I have talked about physics a number of times with Maldacena who shares, with *e.g.* Nambu, exceptional modesty. Administration may nurture his single-processor thinking, a bit reminiscent of Einstein and general relativity? It has been stimulating to write papers about AdS/CFT and, ignoring the admonition that each self-citation counts (-5), here is my list [18].

CONGRATULATIONS TO AdS/CFT ON ITS TENTH ANNIVERSARY

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